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1994 J. Phys. A: Math. Gen. 27 L875

(http://iopscience.iop.org/0305-4470/27/23/002)

J. Phys. A: Math. Gen. 27 (1994) L875–L879. Printed in the UK

LETTER TO THE EDITOR

Persistent fluxon current via the Aharonov–Casher effect in one-dimensional mesoscopic rings: continuum model

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Received 30 August 1994

Abstract. By extending Lieb and Liniger's Bose gas analysis to many hard-core bosons confined to a one-dimensional mesoscopic ring in the presence of Aharonov–Casher flux (AC), persistent fluxon currents are investigated for the first time, and are found to be periodic in the AC flux with period $\Phi_0 = hc/e$. The interesting mesoscopic parity effects due to evenness or oddness of the particle number N are discussed. More importantly, it is demonstrated exactly that, in the absence of the external AC flux, whether a self-sustained fluxon current exists depends only upon the choice of the boundary condition.

Topological effects in quantum-mechanical systems are manifested through the generation of relative phases which accumulate on the wavefunction of a particle moving through a non-simply-connected force-free region. The generic phenomenon of this type is the Aharonov–Bohm (AB) effect, which is due to the presence of a vector potential in the Hamiltonian of the particle [1]. One of the most wonderful demonstrations of the AB effect is the persistent current in a mesoscopic normal-metal ring threaded by magnetic flux $\Phi_{AB} = \oint \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{l}$ [2,3]. In this case, the persistent current itself also produces magnetic flux, in addition to the externally applied flux, and the possibility of a self-sustained (or spontaneous) persistent current for many-fermion systems was recently pointed out [4]. Interestingly, Aharonov and Casher (AC) [5] suggest that a neutral particle with a magnetic moment μ may exhibit a topological force-free interference effect, as a result of an interaction with a charged wire. Consequently, the AB effect admits an electromagnetic duality. This AC effect was discussed for a fluxon in type-II superconductors [6] and a vortex in Josephson-junction arrays [7,8]. In addition, based upon a similar idea for the spontaneous AB effect, an interesting spontaneous AC effect, due to the many-body effect in one-dimensional (1D) mesoscopic normal-metal rings [9,10] and Josephson-junction arrays [9], was recently investigated. In particular, it is noteworthy that the previously mentioned fluxon (or vortex) in type-II superconductors (or Josephson-junction arrays) can be treated as a standard neutral hard-core boson with given mass and magnetic moment when its essential nature is addressed. On the other hand, the hard-core boson spectrum is identical to that of a non-interacting one-component fermion, up to the parity of a finite particle number N. Such finite-size effects are of great importance in mesoscopic systems. In view of these considerations, it seems desirable to investigate, in detail, the parity effects due to evenness and oddness of N. In this paper, by extending Lieb and Liniger's (LL) Bose gas analysis [11] to many hard-core bosons confined to a 1D mesoscopic ring pierced by a charged rod, and with particular emphasis on the parity effects, we shall calculate the total energy and the corresponding persistent fluxon current, which are of primary interest here. More importantly, it is demonstrated exactly that, in the absence of the external AC flux, whether a self-sustained fluxon current exists depends only upon whether the boundary condition is antiperiodic or not.

In the presence of an AC flux, the mechanical momentum of a boson with magnetic moment μ is given by $p-\mathcal{E}\times\mu/c$, where $\mathcal{E}$ is the electric field [5]. It has been demonstrated that the interaction term $\mathcal{E}\times\mu/c$, although appearing as a local interaction, represents a non-local interaction, and the AC effect is essentially non-local in its nature [6]. Therefore, we are able to work with a gauge in which the field does not appear explicitly in the Hamiltonian for many bosons, but enters the calculation via the flux-modified boundary condition. The Schrödinger equation for N bosons, which are confined to a 1D ring with radius R, vanishingly small width a and thickness l, and interact via a δ-function potential, can be written as

$$\left(-\frac{\hbar^2}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2}+2\gamma\sum_{\langle i,j\rangle}\delta(x_i-x_j)\right)\Psi=E\Psi \tag{1}$$

where 2γ is the amplitude of the δ function. The wavefunction Ψ satisfies the flux-modified boundary conditions in each variable, which reads in part:

$$\Psi(x_1+L,x_2,\ldots,x_N)=\exp(\mathrm{i}2\pi f_{\mathrm{AC}})\Psi(x_1,x_2,\ldots,x_N) \tag{2}$$

with a similar condition for the derivatives. In equation (2), $f_{\mathrm{AC}}=\Phi_{\mathrm{AC}}/\Phi_0$ with $\Phi_{\mathrm{AC}}=(\mu/e)\oint\hat{z}\cdot(\mathrm{d}l\times\mathcal{E})$ and $\Phi_0\equiv hc/e$. Following LL's analysis, we can easily find the Bethe ansatz (BA) [12] consistency conditions. To ensure that the BA wavefunction

$$\Psi(x_1,x_2,\ldots,x_N)=\sum_P a(P)P\exp\left(\mathrm{i}\sum_{j=1}^{N}k_jx_j\right)\qquad(x_1<x_2<\cdots<x_N) \tag{3}$$

is an exact solution of equation (1), the phases θ_{sj} must read:

$$\theta_{sj}=-2\tan^{-1}[(\hbar^2/2m\gamma)(k_s-k_j)] \tag{4}$$

$$(-1)^{N-1}\mathrm{e}^{-\mathrm{i}k_jL}=\exp\left(-\mathrm{i}2\pi f_{\mathrm{AC}}+\mathrm{i}\sum_{s=1}^{N}\theta_{sj}\right).$$

The phases θ_{sj} are related to the prefactors $a(P)$. When $\gamma=\infty$, we have the hard-core boson case in which we are interested and all the θ's are zero so that equation (5) leads to

$$\begin{aligned}k&=(2\pi/L)(n+f_{\mathrm{AC}})\qquad\text{for odd }N\\&=(2\pi/L)(n+f_{\mathrm{AC}}+\tfrac{1}{2})\qquad\text{for even }N\end{aligned} \tag{6}$$

where $n=0,\pm1,\pm2,\ldots$, and f_{AC} is in the range $[-\frac{1}{2},\frac{1}{2})$. It can be seen from equation (6) that the spectrum of hard-core bosons being identical to that of non-interacting one-component fermions is true only for finite N, since N is odd. Bearing this fact in mind, we can perform the calculations for the physical quantities as in the case of non-interacting one-component fermions. Therefore, the total energy of N particles can be written as

$$E=\sum_n\left(\frac{\hbar^2k_n^2}{2m}\right). \tag{7}$$

Substituting equation (6) into equation (7), we obtain the energy of the ground state:

$$E = \sum_{n=-n_0}^{n_0} \frac{\hbar^2 n^2}{2mR^2} + \frac{N\hbar^2}{2mR^2} f_{\mathrm{AC}}^2 \tag{8}$$

for $N = 2n_0 + 1$ with $n_0 = 0, 1, \ldots,$ and

$$E = \sum_{n=-n_0}^{n_0-1} \frac{\hbar^2 (n+\frac{1}{2})^2}{2mR^2} + \frac{N\hbar^2}{2mR^2} f_{\mathrm{AC}}^2 \tag{9}$$

for $N = 2n_0$ with $n_0 = 1, 2, \ldots$.

There is a close connection between the states of a fermion-like particle in a loop and the one-dimensional Bloch problem, as seen by identifying $2\pi\Phi_{\mathrm{AC}}/\Phi_0$ and $k(2\pi R)$ with k as the wavevector [13–15]. The energy levels of the ring form microbands as a function of Φ_{AC} with the period Φ_0 analogous to the Bloch bands in the extended k-zone picture. The fluxon current carried by the nth level is

$$I_n = \frac{\phi v_n}{2\pi R} = \frac{\phi}{2\pi\hbar}\frac{\partial E_n}{\partial f_{\mathrm{AC}}} \tag{10}$$

where ϕ is the magnetic flux carried by each hard-core boson. At zero temperature, the fluxon current in the ring for a fixed number of hard-core bosons N should be the sum over the individual contribution from each occupied state, i.e.

$$I = \sum_n I_n = \frac{\phi}{2\pi\hbar}\frac{\partial E}{\partial f_{\mathrm{AC}}} = \frac{N\phi\hbar}{2\pi m R^2} f_{\mathrm{AC}}. \tag{11}$$

Clearly, at $T = 0$ K, the fluxon current is a piecewise periodic function of the AC flux. In each periodic region, I varies linearly with Φ_{AC}, and there are discontinuous jumps when one period Φ_0 is over.

At this stage, we wish to point out that the AC flux Φ_{AC}, which drives the persistent current I, is the sum of the externally applied Φ_{ext} and the flux Φ_I, induced by the persistent current itself, $\Phi_{\mathrm{AC}} = \Phi_{\mathrm{ext}} + \Phi_I$. This raises the possibility of a spontaneous AC effect in the absence of the externally applied flux Φ_{ext}. It is well known that an electric field ε_n is essentially induced when a particle carrying a magnetic flux ϕ moves with velocity $\boldsymbol{v}_n$. According to Faraday's law:

$$\nabla \times \varepsilon_n = -\frac{1}{c}\frac{\partial b}{\partial t}$$

we can obtain $\varepsilon_n = b \times v_n/c = (2\pi R/\phi c) b \times I_n$, with $b = \phi/(2\pi R a)$, as the effective fluxon density of an individual particle. The total electric field generated by the fluxon current *inside the ring* can easily be obtained

$$\mathcal{E} = \sum_n \varepsilon_n = -\left(\frac{I}{ac}\right)\hat{r} \tag{12}$$

where $\hat{r}$ is the unit vector of radial component. The induced AC flux is then found to be

$$f_{\mathrm{AC}} = \frac{\mu \mathcal{E}_r R}{\hbar c} = -\frac{\phi l R}{4\pi a \hbar c^2} I. \tag{13}$$

Here we have used $\mu = \phi l/4\pi$ with l as the effective length of the flux line. In the absence of the external AC flux, equations (11) and (13) lead to a self-consistent solution

$$f_{\mathrm{AC}}^{(s)} = 0. \tag{14}$$

On the other hand, the total energy of the whole system consists of two parts: the energy of particles in the ring E and the energy of the electric field $E_{\mathcal{E}}$, i.e. $E_{\mathrm{T}} = E + E_{\mathcal{E}}$, where

$$E_{\mathcal{E}} = \frac{1}{8\pi}\int \mathcal{E}^2\,\mathrm{d}^3x = \frac{4\pi^2 a\hbar^2 c^2}{\phi^2 l R} f_{\mathrm{AC}}^2. \tag{15}$$

It is obvious that the total energy E_{T} reaches its minimum just at $f_{\mathrm{AC}} = f_{\mathrm{AC}}^{(s)} = 0$, i.e.

$$\left.\frac{\partial E_{\mathrm{T}}}{\partial f_{\mathrm{AC}}}\right|_{s} = 0 \qquad \left.\frac{\partial^2 E_{\mathrm{T}}}{\partial f_{\mathrm{AC}}^2}\right|_{s} > 0$$

which implies that the stable ground state of the system does not carry the spontaneous fluxon current via the AC effect, regardless of whether the number of hard-core bosons N is *even* or *odd*.

It is worthwhile comparing these results for hard-core boson systems with those for other systems.

(i) For normal free bosons, all of them occupy the lowest energy level with zero momentum at zero temperature, and the introduction of a small AC phase will make the energy of the system higher. Therefore, a spontaneous AC effect should not exist in the normal free-boson systems.

(ii) In mesoscopic rings comprised of polarized spin-$\frac{1}{2}$ free fermions, the spontaneous AC effect could exist with an *even* number of particles because of the pairwise cancellation of the currents in the states with $\pm n$ [4].

(iii) In the systems we are discussing, although the behaviour of hard-core bosons is fermion-like, once, however, the periodic boundary condition on the wavefunction is chosen, it is the parity dependence of the pseudo-wavevector k that kills the spontaneous AC effect.

(iv) It is interesting to notice that, if we choose the antiperiodic boundary condition in the absence of the AC flux, $\Psi(x_1 + L, x_2, \ldots, x_N) = -\Psi(x_1, x_2, \ldots, x_N)$, which is not unreasonable when we insert a π-phase-shift junction in the ring, we can easily find a ground state with a spontaneous AC flux

$$f_{\mathrm{AC}}^{(s)} = \left[2 + \frac{16\pi^2 ma Rc^2}{Nl\phi^2}\right]^{-1} \tag{16}$$

which may be observable.

For a large, but not infinite, interaction strength γ, by performing the perturbation treatment on equation (4) to the first order in $(1/\gamma)$, it has been found that the energy spectrum is merely scaled by a factor $(1+N\hbar^2/m\gamma L)^{-2}$, and all the conclusions mentioned above do not change. For a finite γ, the situation becomes complicated and needs further investigation.

Finally, it may be constructive to make some remarks concerning the experimental relevance of the AC effect in 1D hard-core boson systems. Consider an annular type-II superconducting sample in the dilute mixed state with $a \sim \lambda \sim \xi$ with the external magnetic field applied along the z-axis, where λ and ξ are the penetration and the superconducting coherence lengths, respectively. This setup ensures that the physical system effectively behaves one-dimensionally and exhibits the discussed AC effect. In the ideal case, and at very low temperature, the possible maximum signal of the radial voltage due to the spontaneous AC effect is roughly estimated as

$$V_{\mathrm{AC}}^{(\mathrm{s})} \sim \mathcal{E}^{(\mathrm{s})}\lambda \sim \frac{e}{R}\left(\frac{\lambda}{l}\right) \sim 10^{-6}\ (\mathrm{V})$$

if $R \sim 10\ \mu$m and $l \sim 10\lambda$ are chosen, which may be observable. Therefore, the results presented here provide a strong motivation to design and carry out such measurements of the AC effect.

The authors would like to acknowledge Professor Jinming Dong for helpful discussions. This work was supported by a RGC grant of Hong Kong and a CRCG research grant at the University of Hong Kong.

References

[1] Aharonov Y and Bohm D 1959 *Phys. Rev.* **115** 485
[2] Landauer R and Büttiker M 1985 *Phys. Rev. Lett.* **54** 2049
Cheung H-F, Gefen Y, Riedel E K and Shih W H 1988 *Phys. Rev.* B **37** 6050
[3] Lévy L P, Dolan G, Dunsmir J and Bouchiat H 1990 *Phys. Rev. Lett.* **64** 2074
Chandrasekhar V, Webb R A, Brady M J, Ketchen M B, Gallagher W J and Kleinsaser A 1991 *Phys. Rev. Lett.* **67** 3528
[4] Wohlleben D, Esser M, Freche P, Zipper E and Szopa M 1991 *Phys. Rev. Lett.* **66** 3191; 1992 *Mod. Phys. Lett.* B **6** 1481
[5] Aharonov Y and Casher A 1984 *Phys. Rev. Lett.* **53** 319
Hagen C R 1990 *Phys. Rev. Lett.* **64** 2347
[6] Reznik R and Aharonov Y 1989 *Phys. Rev.* D **40** 4178
[7] van Wees B J 1990 *Phys. Rev. Lett.* **65** 255
Orlando T P and Delin K A 1991 *Phys. Rev.* B **43** 8717
[8] Elion W J, Wachers J J, Sohn L L and Mooij J E 1993 *Phys. Rev. Lett.* **71** 2311
[9] Choi M Y 1993 *Phys. Rev. Lett.* **71** 2987
[10] Zhu Jian-Xin and Wang Z D 1994 *J. Phys.: Condens. Matter* **6** L329
[11] Lieb E H and Liniger W 1963 *Phys. Rev.* **130** 1605
[12] Bethe H 1931 *Z. Phys.* **31** 205
[13] Büttiker M, Imry Y and Landauer R 1983 *Phys. Lett.* **96A** 365
[14] Byers N and Yang C N 1961 *Phys. Rev. Lett.* **7** 46
[15] Bloch F 1970 *Phys. Rev.* B **2** 109